The Superconductor-Insulator Transition in a Tunable Dissipative Environment

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We study the influence of a tunable dissipative environment on the dynamics of Josephson junction arrays near the superconductor-insulator transition. The experimental realization of the environment is a two dimensional electron gas coupled capacitively to the array. This setup allows for the well-controlled tuning of the dissipation by changing the resistance of the two dimensional electron gas. The capacitive coupling cuts off the dissipation at low frequencies. We determine the phase diagram and calculate the temperature and dissipation dependence of the array conductivity. We find good agreement with recent experimental results.

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Quantum phase transitions attract intense attention because of their paradigmatic nature: they are relevant to a host of experimental issues. Examples include the superconductor-insulator transition in granular superconductors [1], the transition between Quantum Hall states [2], transitions in disordered magnets [3], and the physics of vortices in the presence of columnar disorder [4]. Josephson junction arrays constitute a particularly attractive experimental testing ground for the superconductor-insulator (SI) transition, because all parameters are well under control, and are widely tunable [5,6]. In these systems the SI transition can be driven by quantum fluctuations when the charging energy E_C becomes comparable to the Josephson coupling energy E_J [7]. It was understood early that dissipation is also capable of driving an SI transition. The phase diagram of a single Josephson junction in a dissipative environment was explored by Schmid [8]. Strong dissipation suppresses quantum fluctuations and restores the classical behaviour with a finite supercurrent. For weak damping, however, quantum fluctuations suppress the supercurrent to zero. When an array is built from the junctions, at strong dissipation phase fluctuations are again damped, favouring phase coherence and global superconductivity. This type of SI transition is present in arrays of Josephson junctions as well as in thin films [9–12].

The experimental verification of a dissipation tuned superconductor-insulator transition is still open. The actual strength of the dissipation is hard to control. An indicator may be the normal state resistance, although it is unclear how this translates into a dissipation below the bulk transition temperature, where the opening of a gap freezes out the gapless excitations [13]. It is also unsettled whether the dissipation or the Coulomb interaction is the main driving force for the transition. Recently the Berkeley group succeeded to fabricate and investigate Josephson junction arrays with tunable dissipation by placing a Josephson junction array on top of a two dimensional electron gas (2DEG), separated by an insulator [6]. The electron density and sheet resistance of the 2DEG are varied by tuning a gate voltage, without influencing the other parameters of the array. The main result is that the array resistance exhibits a temperature dependence, parametrized by the dissipation, which is reminiscent of a superconductor-insulator transition [1,5].

In the present work we model the experimental setup by an array capacitively coupled to a 2DEG. Our results for the array resistance per square $R(T)$ as a function of temperature T and 2DEG resistance $R_{\rm 2DEG}$ track the experimental data well. The T dependence of $R(T)$ is characteristic of an imminent SI transition tuned by $R_{\rm 2DEG}$. However, at the lowest temperatures it exhibits a sharp reentrant rise. In our model this is naturally explained by the presence of a cutoff in the spectrum of the dissipation at low frequencies.

The quantum-dynamical variables describing a Josephson junction array are the phases φ_j of the superconducting order parameter on island j. The dynamics of the 2DEG is formulated in terms of a fluctuating scalar potential $V(\mathbf{r},t)$ [13]. This potential can, in close analogy to the Josephson relation, be represented by a phase-variable $\phi(\mathbf{r},t)$, defined by $\hbar\dot{\phi}(\mathbf{r},t) = 2eV(\mathbf{r},t)$. For the coupled system the action takes the form

$$S[\varphi,\phi] = S_{\rm JJA}[\varphi] + S_{\rm I}[\varphi,\phi] + S_{\rm 2DEG}[\phi]\,. \tag{1}$$

The array is characterized by the Josephson coupling E_J and the inter-grain capacitance C_1, which represents the bare, unscreened Coulomb interaction in 2 dimensions

$$S_{\rm JJA}[\varphi] = \frac{1}{2}\int_{k,\omega_\mu} \frac{C_1}{4e^2}\gamma(k)\omega_\mu^2|\varphi_{k,\omega_\mu}|^2 - E_J\int d\tau \sum_{<ij>}\cos(\varphi_i-\varphi_j)\,, \tag{2}$$

where $\gamma(k)$ is, on a square 2d lattice, given by $\gamma(k) = 4-2\cos(ak_x)-2\cos(ak_y)$, a is the lattice constant, chosen as the unit of length. At the relevant long wavelengths $\gamma(k)\approx k^2$. Coupling to the 2DEG is characterized by a capacitance C_0

$$S_{\rm I}[\varphi,\phi] = \frac{1}{2}\int_{k,\omega_\mu} \frac{C_0\omega_\mu^2}{4e^2}|\varphi_{k,\omega_\mu}-\phi_{k,\omega_\mu}|^2\,, \tag{3}$$

where $\dot{\varphi} - \dot{\phi}$ is the potential *difference* between the array and the 2DEG. This formulation of an interaction mediated by a local capacitance correctly represents the electrostatics of the system. The interaction between the charges in the array and the 2DEG is determined by the *inverse* capacitance matrix which represents a long-range interaction of charges.

The dynamics of the 2DEG is ohmic, with resistance R_{2DEG}. The microscopic details of the 2DEG do not play a role on the length scales considered presently. The corresponding action is

$$S_{\mathrm{2DEG}}[\phi] = \frac{1}{2}\int_{k,\omega_\mu} \frac{R_Q}{2\pi R_{\mathrm{2DEG}}} k^2|\omega_\mu||\phi_{k,\omega_\mu}|^2 \,. \qquad (4)$$

The scale of resistance is set by its natural quantum unit, $R_Q = h/(4e^2)$. The interactions within the 2DEG, and its diffusive behaviour influence the action only at higher momenta and frequencies.

The effective action for the array is constructed by integrating out ϕ, the fluctuating voltage of the 2DEG

$$\begin{aligned} S_{\mathrm{eff}}[\varphi] = \frac{1}{2}\int_{k,\omega_\mu} & D_0^{-1}(k,\omega_\mu)|\varphi_{k,\omega_\mu}|^2 \\ & -E_J\int d\tau \sum_{<ij>} \cos(\varphi_i - \varphi_j) \quad . \end{aligned} \qquad (5)$$

The propagator for the φ reads

$$D_0^{-1}(k,\omega_\mu) = \frac{C_1}{4e^2}k^2\omega_\mu^2 + \frac{C_0}{4e^2}\frac{k^2\omega_\mu^2}{k^2 + |\omega_\mu|/\Omega_0} \quad , \qquad (6)$$

where $1/\Omega_0 = R_{\mathrm{2DEG}}C_0$. The dynamics of the phases φ, described by the propagator D_0, has three characteristic frequency regimes:

1) In the limit of small frequencies, $\omega < \Omega_0$, dissipation is frozen out, and the dynamics of the phase is capacitive. In this limit Eq.(6) reduces to $D_0^{-1} = \omega_\mu^2(C_1k^2 + C_0)/(4e^2)$. The 2DEG screens the electrostatic interaction in the array beyond the characteristic length scale $\Lambda = \sqrt{C_1/C_0}$.

2) At frequencies exceeding Ω_0, the resistivity of the 2DEG induces damping for the superconducting phase. The origin of this damping is that the voltage fluctuations of the 2DEG cannot follow the fluctuations of φ adiabatically. This creates damping with a strength determined by the 2DEG resistance. From Eq.(6) one finds $D_0^{-1} = C_1k^2\omega_\mu^2/(4e^2) + k^2|\omega_\mu|R_Q/(2\pi R_{\mathrm{2DEG}})$, (for $\omega > \Omega_0$), which describes an array of resistively shunted junctions [10].

3) At even higher frequencies $\omega_\mu \gg \Omega_1 = 1/(R_{\mathrm{2DEG}}C_1)$, the response is again capacitive, but now determined by the inter-grain capacitance C_1. The leading frequency dependence of the propagator of Eq.(6) is $D_0^{-1} = \omega_\mu^2 k^2 C_1/(4e^2)$.

In sum, the effective action for the array is ohmic only in an *intermediate* frequency range $\Omega_0 < \omega_\mu < \Omega_1$. At the lowest and highest frequencies the dynamics is capacitive. The two energy scales are well separated in the case $C_0 \gg C_1$. A quantum phase transition is driven by the behaviour of the action at the lowest frequencies. In the present case - as the dissipative action is cut off at the lowest frequencies - a dissipation driven transition cannot occur in the strict sense. However, a quasi-critical behaviour can be observed at temperatures and voltages exceeding the low energy scale Ω_0. In the limit $\Omega_0 \to 0$ ($C_0 \to \infty$) this behaviour converges to the true dissipation-tuned transition.

To characterize this quasi-critical behaviour, we now evaluate the electromagnetic response of the array at finite temperatures. The array conductivity, as a function of Matsubara frequencies, is calculated via the Kubo formula. In the regime where the Josephson energy E_J is smaller than the capacitive energy scale $E_C = e^2/(2C_0)$, insight can be gained by a perturbative expansion to second order in E_J. For the longitudinal part of the conductivity we obtain

$$\sigma_{xx}(\omega_\nu) = \frac{2e^2E_J^2}{\hbar}\int_0^\beta d\tau \frac{1 - \mathrm{e}^{i\omega_\nu\tau}}{\omega_\nu} g(\tau) \,, \qquad (7)$$

where $g(\tau) = \langle \cos[\varphi(0,\tau) - \varphi(\hat{x},\tau) - \varphi(0) + \varphi(\hat{x},0)] \rangle_0$. The correlation function $g(\tau)$ depends on $\hat{x}$, which is defined to connect the nearest neighbors in the x direction. The expectation value $\langle ... \rangle_0$ is taken with the action $S_{\mathrm{eff}}[\varphi]$ at $E_J = 0$. The result is

$$\begin{aligned} g(\tau) &= \exp\left\{ \frac{1}{\beta}\sum_\mu (1 - \cos\omega_\mu\tau)\, d(\omega_\mu) \right\} \cdot g_W(\tau) \\ d(\omega_\mu) &= \int \frac{d^2k}{4\pi^2}(2 - 2\cos k_x) D_0(k,\omega_\mu) \\ &\approx \frac{2\pi}{\alpha|\omega_\mu|}\frac{1}{1 + |\omega_\mu|/\Omega_1} + \frac{E_0}{\omega_\mu^2} \quad , \end{aligned} \qquad (8)$$

where $\alpha = R_Q/R_{\mathrm{2DEG}}$, $E_0 = 2\pi\Omega_0/\alpha$, and $g_W(\tau)$ represents a summation over the winding numbers, which reflects the discrete nature of the charge transfer in the array [14]. In accordance with the above, $d(\omega_\mu)$ represents ohmic damping in the intermediate frequency range $\Omega_0 < \omega_\mu < \Omega_1$. The lattice structure and the range of the electrostatic interaction influence the precise value of the damping strength α upto a multiplicative constant c. For a square array and short range interactions ($C_0 \gg C_1$) this prefactor is $c \sim \mathcal{O}(1)$.

The conductivity as a function of real frequencies follows by analytic continuation, $\sigma(\omega) = \sigma_{xx}(\omega_\mu \to -i\omega + \delta)$

$$\sigma(\omega) = \frac{2\pi E_J^2}{R_Q}\int_0^\infty dt \frac{1 - \mathrm{e}^{i\omega t}}{-i\omega}\mathrm{Im}[g(it)] \qquad (9)$$

The analytic continuation of $g(\tau)$ reads

$$g(it) = \exp\Bigg(-\frac{2}{\alpha}\int_0^\infty d\omega \Big[\frac{1 - \cos\omega t}{\omega(1 + \omega^2/\Omega_1^2)} \coth\frac{\beta\omega}{2}$$

$$-i\frac{\sin\omega t}{\omega(1+\omega^2/\Omega_1^2)}\Big] - iE_0 t\Big) \cdot \langle\cos(ntE_0)\rangle_n \,. \quad (10)$$

Here $\langle ...\rangle_n$ is taken with the action $S_n = \beta E_0 n^2/2$, n integer. We identify the energy scale E_0 with the Mott gap, the energy cost of adding on extra charge to the array. The Mott gap E_0 is considerably reduced by random offset charges and, close to the SI transition, also by charge fluctuations [15]. For low temperatures $T \ll \Omega_1$ the correlator is given by

$$\begin{aligned}\mathrm{Im}\, g(it) = &\left(\frac{1-\mathrm{e}^{-2\pi t/\beta}}{2\pi t/\beta}\right)^{-\frac{2}{\alpha}} \sin\left(\frac{\pi}{\alpha}(1-\mathrm{e}^{-\Omega_1 t}) + E_0 t\right) \\ &\exp\Big[-\frac{1}{\alpha}\Big(2\gamma + 2\ln(\Omega_1 t) + \mathrm{e}^{\Omega_1 t}\mathrm{E}_1(\Omega_1 t) \\ &-\mathrm{e}^{-\Omega_1 t}\mathrm{Ei}(\Omega_1 t)\Big) - \frac{2\pi t}{\alpha\beta}\Big]\langle\cos(ntE_0)\rangle_n \,, \quad (11)\end{aligned}$$

where $\gamma = 0.577...$ is Euler's constant, Ei and E_1 are exponential integrals. A subsequent numerical integration directly gives the conductivity as shown in Fig.1, for various values of R_{2DEG}. The conductivity $\sigma(T)$ in the *intermediate* temperature range $\Omega_0 < T < \Omega_1$ behaves as $\sim T^{2/\alpha-2}$, in analogy to the single junction result [8,10]. For $\alpha > 1$ the decrease of $R(T)$ indicates an impending SI transition. However, since the dissipation is frozen out below Ω_0, $R(T)$ rises sharply, eventually becoming an insulator at $T = 0$. For $\alpha < 1$, $R(T)$ is monotonously increasing with decreasing T.

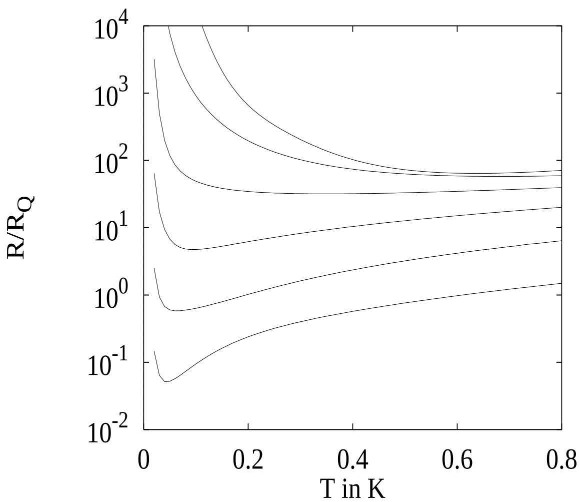

FIG. 1. Temperature dependence of the array resistivity $1/\sigma(\omega{=}0)$. E_0=0.2 K, E_J=0.28 K, C_0/C_1=10. The dissipation takes the values α=20 (lowest curve), 5, 2, 1, 0.5, 0.25 (uppermost curve).

These perturbative results can be interpreted as a renormalization of E_J by the dissipative processes. Writing $\sigma(T) \sim (E_J^{\mathrm{ren}})^2$ identifies $E_J^{\mathrm{ren}} \sim T^{1/\alpha-1}$. This renormalization stops at $\sim \Omega_0$. At Ω_0 the model is equivalent to an XY model, with renormalized parameters. This model has a phase transition at $E_J^{\mathrm{ren}}/E_C \approx 1$. Therefore, the SI phase boundary is given by $E_J/E_C \approx (C_1/C_0)^{1-1/\alpha}$. With decreasing $\Omega_0 \to 0$ ($C_0 \to \infty$) a true dissipation driven phase transition is approached at $\alpha = 1$. For small α, the dissipative scaling is taken over by the XY scaling [12]. The phase boundary flattens and reaches smoothly its $\alpha = 0$ value of $\mathcal{O}(1)$ (Fig.2). For large E_J we recall the results of Ref. [9], which established that in this limit the effective action reduces to that of a single junction. Consequently, the renormalization group flows, obtained perturbatively at small E_J are characterized by the same power laws at large E_J. Thus in this regime for large α the resistivity decreases monotonically and the array becomes truly superconducting. For small α the RG flows are not completely clear. In the very large E_J limit one expects the dominance of single junction physics at intermediate scales, accompanied with a rise of the resistance. At lower T collective processes drive the system superconducting, manifesting themselves in a sharp drop of the resistivity. The phase diagram and the temperature dependences of the resistance in the four regimes are summarized in Fig.2.

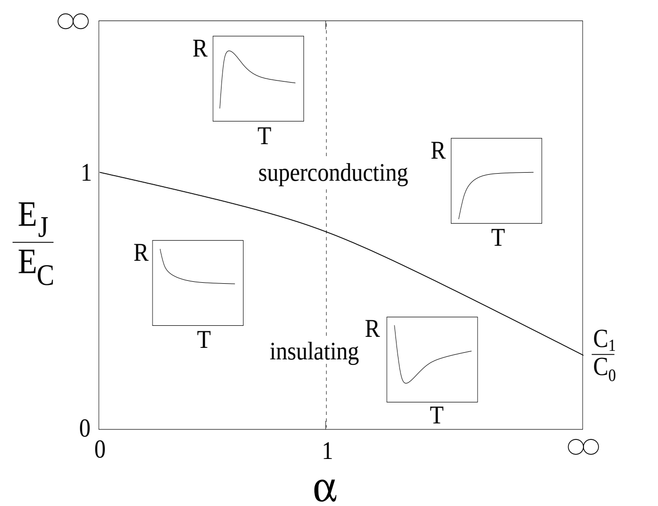

FIG. 2. Phase diagram of an array coupled capacitively to a 2DEG. The insets show $R(T)$ as a function of the temperature in the different regions.

Our perturbative analysis concentrated on the low temperature behaviour of the resistivity. It does not include quasiparticle currents. The normal state resistance R_N at higher temperatures can be only reproduced by including *thermally activated* quasiparticles. They form a parallel channel to the flow of the Cooper pairs. Using the standard BCS gap $\Delta(T)$ introduces visible change only close to the bulk T_{c0}. However, it was recently argued that phase-space considerations seriously *reduce* the gap, experienced by the quasiparticles [16]. In Fig.3, $R(T)$ is plotted with such a parallel normal channel, using a reduced gap value of $E_g = 0.2$K. $R(T)$ now exhibits a convergence to the normal state resistivity R_N at higher temperatures.

Recent experiments carefully explored Josephson junc-

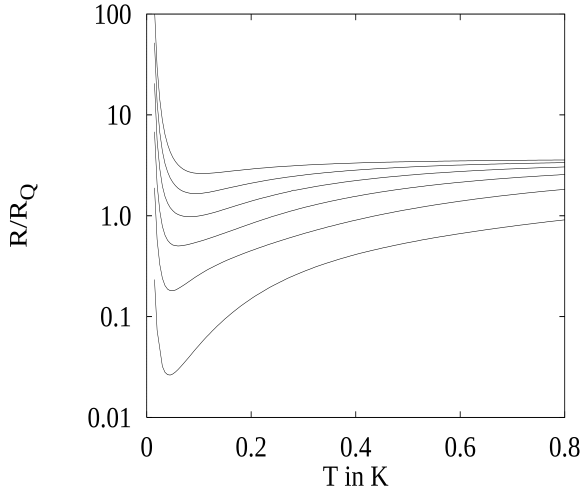

FIG. 3. Array resistivity $1/\sigma(\omega{=}0)$ with a parallel thermally activated channel $R_N \exp(E_g/T)$, with $E_g{=}0.2\,\mathrm{K}$. $E_0{=}0.2\,\mathrm{K}$, $E_J{=}0.28\,\mathrm{K}$, $C_0/C_1{=}10$, and $R_N{=}23\,\mathrm{k}\Omega$. At $R_{2\mathrm{DEG}}{=}$ 200, 700, 1200, 1700, 2200, 2700 Ω.

tion arrays capacitively coupled to a 2DEG [6]. The experiments fall in the parameter regime, where E_J/E_C is small and our perturbative analysis is applicable. The temperature dependence of the resistivity $R(T)$ is strikingly similar to that in Fig.3. Also, the dependence of the array resistance on $R_{2\mathrm{DEG}}$ at fixed temperatures was determined. Since $R_Q/R_{2\mathrm{DEG}} \sim \alpha$, the power law dependence $R(T) \sim T^{2-2/\alpha}$ translates into an exponential relation between R and $R_{2\mathrm{DEG}}$ (Fig.4). This is again in good agreement with the experiments [6].

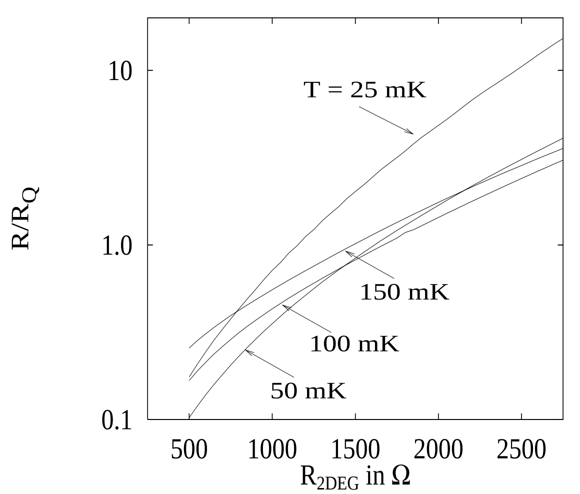

FIG. 4. Array resistivity $1/\sigma(\omega{=}0)$ as a function of $R_{2\mathrm{DEG}}$. $E_0{=}0.2\,\mathrm{K}$, $E_J{=}0.28\,\mathrm{K}$, $C_0/C_1{=}10$, at $T{=}25$, 50, 100, 150 mK.

Lastly, let us consider an Ohmic coupling between the array and the 2DEG. In this case the effective action takes the form of the resistively shunted junction model. The resistances of the 2DEG and the Ohmic shunt between an island and the 2DEG are in series. Since now the spectrum is Ohmic down to zero frequency, the dissipation drives a *true* SI transition. This may be realized by doping the semiconducting layer which separates the array from the 2DEG. In this arrangement a local damping of the phase is generated via the Andreev process, which allows Cooper pairs to decay into normal electrons in the substrate [12,17].

In summary, we developed a model for a Josephson junction array capacitively coupled to a two dimensional electron gas. We determined the phase diagram and the corresponding dependence of the resistivity on the temperature and the resistivity of the 2DEG. Our results compare well to the recent experimental data of Ref. [6]. We also suggested further experiments to investigate the dissipation-tuned phase transition more closely.

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